In general, even if one of the involved functions ($g(x)$ or $G(\omega)$) is real-valued (which is usually the case for physical signals $g(x)$), the other function is complex-valued. One may also note that the forward transformation $\mathcal{F}$ (Eqn. (13.20)) and the inverse transformation $\mathcal{F}^{-1}$ (Eqn. (13.21)) are almost completely symmetrical, the sign of the exponent being the only difference.[6] The spectrum produced by the Fourier transform is a new representation of the signal in a space of frequencies. Apparently, this "frequency space" and the original "signal space" are *dual* and interchangeable mathematical representations.

13.1.5 Fourier Transform Pairs

The relationship between a function $g(x)$ and its Fourier spectrum $G(\omega)$ is unique in both directions: the Fourier spectrum is uniquely defined for a given function, and for any Fourier spectrum there is only one matching signal—the two functions $g(x)$ and $G(\omega)$ constitute a "transform pair",

$$g(x) \multimap G(\omega).$$

Table 13.1 lists the transform pairs for some selected analytical functions, which are also shown graphically in Figs. 13.3 and 13.4.

Table 13.1
Fourier transforms of selected analytical functions; $\delta()$ denotes the "impulse" or *Dirac* function (see Sec. 13.2.1).

Function	*Transform Pair* $g(x) \multimap G(\omega)$	*Figure*
Cosine function with frequency ω_0	$g(x) = \cos(\omega_0 x)$ $G(\omega) = \sqrt{\frac{\pi}{2}} \cdot \big(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\big)$	13.3 (a, c)
Sine function with frequency ω_0	$g(x) = \sin(\omega_0 x)$ $G(\omega) = \mathrm{i}\sqrt{\frac{\pi}{2}} \cdot \big(\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\big)$	13.3 (b, d)
Gaussian function of width σ	$g(x) = \frac{1}{\sigma} \cdot e^{-\frac{x^2}{2\sigma^2}}$ $G(\omega) = e^{-\frac{\sigma^2\omega^2}{2}}$	13.4 (a, b)
Rectangular pulse of width $2b$	$g(x) = \Pi_b(x) = \begin{cases} 1 & \text{for } \lvert x\rvert \leq b \\ 0 & \text{otherwise} \end{cases}$ $G(\omega) = \frac{2b\sin(b\omega)}{\sqrt{2\pi}\omega}$	13.4 (c, d)

The Fourier spectrum of a *cosine function* $\cos(\omega_0 x)$, for example, consists of two separate thin pulses arranged symmetrically at a distance ω_0 from the origin (Fig. 13.3 (a, c)). Intuitively, this corresponds to our physical understanding of a spectrum (e. g., if we think of a pure

[6] Various definitions of the Fourier transform are in common use. They are contrasted mainly by the constant factors outside the integral and the signs of the exponents in the forward and inverse transforms, but all versions are equivalent in principle. The symmetric variant shown here uses the same factor $(1/\sqrt{2\pi})$ in the forward and inverse transforms.