$$w_{\rm crm}(x) = w_{\rm cs}(x, 0.5, 0)$$
$$= \frac{1}{2} \cdot \begin{cases} 3 \cdot |x|^3 - 5 \cdot |x|^2 + 2 & \text{for } 0 \le |x| < 1 \\ -|x|^3 + 5 \cdot |x|^2 - 8 \cdot |x| + 4 & \text{for } 1 \le |x| < 2 \\ 0 & \text{for } |x| \ge 2\,. \end{cases} \quad (16.57)$$

Examples of signals interpolated with this kernel are shown in Fig. 16.22 (a–c). The results are similar to ones produced by cubic interpolation (with $a = 1$, see Fig. 16.20) with regard to sharpness, but the Catmull-Rom reconstruction is clearly superior in smooth signal regions (compare, e. g., Fig. 16.20 (d) vs. Fig. 16.22 (a)).

Cubic B-spline approximation

With parameters set to $a = 0$ and $b = 1$, Eqn. (16.56) corresponds to a cubic B-spline function [10] of the form

$$w_{\rm cbs}(x) = w_{\rm cs}(x, 0, 1)$$
$$= \frac{1}{6} \cdot \begin{cases} 3 \cdot |x|^3 - 6 \cdot |x|^2 + 4 & \text{for } 0 \le |x| < 1 \\ -|x|^3 + 6 \cdot |x|^2 - 12 \cdot |x| + 8 & \text{for } 1 \le |x| < 2 \\ 0 & \text{for } |x| \ge 2\,. \end{cases} \quad (16.58)$$

This function is positive everywhere and, when used as an interpolation kernel, causes a pure smoothing effect similar to a Gaussian smoothing filter (see Fig. 16.22 (d–f)). Notice also that—in contrast to all previously described interpolation methods—the reconstructed function does *not* pass through all discrete sample points. Thus, to be precise, the reconstruction with cubic B-splines is not called an *interpolation* but an *approximation* of the signal.

Mitchell-Netravali approximation

The design of an optimal interpolation kernel is always a trade-off between high bandwidth (sharpness) and good transient response (low ringing). Catmull-Rom interpolation, for example, emphasizes high sharpness, whereas cubic B-spline interpolation blurs but creates no ringing. Based on empirical tests, Mitchell and Netravali [72] proposed a cubic interpolation kernel as described in Eqn. (16.56) with parameter settings $a = \frac{1}{3}$ and $b = \frac{1}{3}$, and the resulting interpolation function

$$w_{\rm mn}(x) = w_{\rm cs}\big(x, \tfrac{1}{3}, \tfrac{1}{3}\big)$$
$$= \frac{1}{18} \cdot \begin{cases} 21 \cdot |x|^3 - 36 \cdot |x|^2 + 16 & \text{for } 0 \le |x| < 1 \\ -7 \cdot |x|^3 + 36 \cdot |x|^2 - 60 \cdot |x| + 32 & \text{for } 1 \le |x| < 2 \\ 0 & \text{for } |x| \ge 2. \end{cases} \quad (16.59)$$